TABLE OF CONTENTS
DESIGN EXAMPLES

NOTATION

9.0 INTRODUCTION

9.1 DESIGN EXAMPLE - AASHTO BOX BEAM, BIII-48, SINGLE SPAN WITH NON-COMPOSITE WEARING SURFACE. DESIGNED IN ACCORDANCE WITH AASHTO STANDARD SPECIFICATIONS.

9.2 DESIGN EXAMPLE - AASHTO BOX BEAM, BIII-48, SINGLE SPAN WITH NON-COMPOSITE WEARING SURFACE. DESIGNED IN ACCORDANCE WITH AASHTO LRFD SPECIFICATIONS.

9.3 DESIGN EXAMPLE - AASHTO-PCI BULB-TEE, BT-72, SINGLE SPAN WITH COMPOSITE DECK. DESIGNED IN ACCORDANCE WITH AASHTO STANDARD SPECIFICATIONS.

9.4 DESIGN EXAMPLE - AASHTO-PCI BULB-TEE, BT-72, SINGLE SPAN WITH COMPOSITE DECK. DESIGNED IN ACCORDANCE WITH AASHTO LRFD SPECIFICATIONS.

9.5 DESIGN EXAMPLE - AASHTO-PCI BULB-TEE, BT-72, THREE-SPAN WITH COMPOSITE DECK (MADE CONTINUOUS FOR LIVE LOAD). DESIGNED IN ACCORDANCE WITH AASHTO STANDARD SPECIFICATIONS.

9.6 DESIGN EXAMPLE - AASHTO-PCI BULB-TEE, BT-72, THREE-SPAN WITH COMPOSITE DECK (MADE CONTINUOUS FOR LIVE LOAD). DESIGNED IN ACCORDANCE WITH AASHTO LRFD SPECIFICATIONS.

9.7 DESIGN EXAMPLE - PRECAST CONCRETE STAY-IN-PLACE DECK PANEL SYSTEM. DESIGNED IN ACCORDANCE WITH AASHTO STANDARD SPECIFICATIONS.

9.8 DESIGN EXAMPLE - PRECAST CONCRETE STAY-IN-PLACE DECK PANEL SYSTEM. DESIGNED IN ACCORDANCE WITH AASHTO LRFD SPECIFICATIONS.

Note: Each design example contains a thorough table of contents.

A	= cross-sectional area of the precast beam or section	[STD], [LRFD]
A	= effective tension area of concrete surrounding the flexural tension reinforcement and having the same centroid as the reinforcement divided by the number of bars	[STD], [LRFD]
A_b	= area of an individual bar	[LRFD]
A_c	= total area of the composite section	
A_c	= area of concrete on the flexural tension side of the member	[LRFD]
A_{cv}	= area of concrete section resisting shear transfer	[LRFD]
A_o	= area enclosed by centerlines of the elements of the beam	[LRFD]
A_{ps}	= area of pretensioning steel	[LRFD]
A_{PT}	= transverse post-tensioning reinforcement	
A_s	= area of non-pretensioning tension reinforcement	[STD]
A_s	= area of non-pretensioning tension reinforcement	[LRFD]
A_s	= total area of vertical reinforcement located within the distance (h/5) from the end of the beam	[LRFD]
A_{sf}	= steel area required to develop the ultimate compressive strength of the overhanging portions of the flange	[STD]
A_{sr}	= steel area required to develop the compressive strength of the web of a flanged section	[STD]
A^*_s	= area of pretensioning steel	[STD]
A'_s	= area of compression reinforcement	[LRFD]
A_v	= area of web reinforcement	[STD]
A_v	= area of transverse reinforcement within a distance 's'	[LRFD]
A_{vf}	= area of shear-friction reinforcement	[LRFD]
A_{vh}	= area of web reinforcement required for horizontal shear	
$A_{v\text{-}min}$	= minimum area of web reinforcement	
a	= depth of the compression block	[STD]
a	= distance from the end of beam to drape point	
a	= depth of the equivalent rectangular stress block	[LRFD]
b	= effective flange width	
b	= width of beam	[STD]
b	= width of bottom flange of the beam	
b	= width of the compression face of a member	[LRFD]
b'	= width of web of a flanged member	[STD]
b_e	= effective web width of the precast beam	
b_v	= width of cross section at the contact surface being investigated for horizontal shear	[STD]
b_v	= effective web width	[LRFD]
b_v	= width of interface	[LRFD]
b_w	= web width	[LRFD]
CR_c	= loss of pretension due to creep of concrete	[STD]
CR_s	= loss of pretension due to relaxation of pretensioning steel	[STD]
c	= distance from the extreme compression fiber to the neutral axis	[LRFD]
c	= cohesion factor	[LRFD]
D	= dead load	[STD]
D	= strand diameter	[STD]

DC	= dead load of structural components and non structural attachments	[LRFD]
DFD	= distribution factor for deflection	
DFM	= distribution factor for bending moment	
DF_m	= live load distribution factor for moment	
DFV	= distribution factor for shear force	
DW	= load of wearing surfaces and utilities	[LRFD]
d	= distance from extreme compressive fiber to centroid of the pretensioning force	[STD]
d_b	= nominal strand diameter	[LRFD]
d_c	= thickness of concrete cover measured from extreme tension fiber to center of the closest bar	[STD], [LRFD]
d_e	= distance from exterior web of exterior beam and the interior side of curb or traffic barrier	[LRFD]
d_e	= effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement	[LRFD]
d_p	= distance from extreme compression fiber to the centroid of the pretensioning tendons	[LRFD]
d_v	= effective shear depth	[LRFD]
E	= width of slab over which a wheel load is distributed	[STD]
E_c	= modulus of elasticity of concrete	[STD]
E_c	= modulus of elasticity of concrete	[LRFD]
E_{ci}	= modulus of elasticity of the beam concrete at transfer	
E_p	= modulus of elasticity of pretensioning tendons	[LRFD]
ES	= loss of pretension due to elastic shortening	[STD]
E_s	= modulus of elasticity of pretensioning reinforcement	[STD]
E_s	= modulus of elasticity of reinforcing bars	[LRFD]
e	= eccentricity of the strands at h/2	
e	= eccentricity of strands at transfer length	
e'	= difference between eccentricity of pretensioning steel at midspan and end span	
e_c	= eccentricity of the strand at the midspan	
e_e	= eccentricity of pretensioning force at end of beam	
e_g	= distance between the centers of gravity of the beam and the slab	[LRFD]
F_b	= allowable tensile stress in the precompressed tensile zone at service loads	
F_{pi}	= total force in strands before release	
F_ε	= reduction factor	[LRFD]
f_b	= concrete stress at the bottom fiber of the beam	
f'_c	= specified concrete strength at 28 days	[STD]
f'_c	= specified compressive strength at 28 days	[LRFD]
f_{cdp}	= change of stresses at center of gravity of prestress due to permanent loads, except dead load acting at the time the prestress force is applied (at transfer), calculated at the same section as f_{cgp}	[LRFD]
f_{cds}	= concrete stress at the center of gravity of the pretensioning steel due to all dead loads except the dead load present at the time the pretensioning force is applied	[STD]
f_{cir}	= average concrete stress at the center of gravity of the pretensioning steel due to pretensioning force and dead load of beam immediately after transfer	[STD]
f'_{ci}	= concrete strength at release	[STD]

f'_{ci}	= specified compressive strength of concrete at time of initial loading or pretensioning	[LRFD]
f_{cgp}	= concrete stress at the center of gravity of pretensioning tendons, due to pretensioning force at transfer and the self-weight of the member at the section of maximum positive moment	[LRFD]
f_d	= stress due to unfactored dead load, at extreme fiber of section where tensile stress is caused by externally applied loads	[STD]
f_{pb}	= compressive stress at bottom fiber of the beam due to prestress force	
f_{pc}	= compressive stress in concrete (after allowance for all pretension losses) at centroid of cross section resisting externally applied loads	[STD]
f_{pc}	= compressive stress in concrete after all prestress losses have occurred either at the centroid of the cross section resisting live load or at the junction of the web and flange when the centroid lies in the flange. In a composite section, f_{pc} is the resultant compressive stress at the centroid of the composite section, or at the junction of the web and flange when the centroid lies with in the flange, due to both prestress and to the bending moments resisted by the precast member acting alone	[LRFD]
f_{pe}	= compressive stress in concrete due to effective pretension forces only (after allowance for all pretension losses) at extreme fiber of section where tensile stress is caused by externally applied loads	[STD]
f_{pe}	= effective stress in the pretensioning steel after losses	[LRFD]
f_{pi}	= initial stress immediately before transfer	
f_{po}	= stress in the pretensioning steel when the stress in the surrounding concrete is zero	[LRFD]
f_{ps}	= average stress in pretensioning steel at the time for which the nominal resistance of member is required	[LRFD]
f_{pt}	= stress in pretensioning steel immediately after transfer	[LRFD]
f_{pu}	= specified tensile strength of pretensioning steel	[LRFD]
f_{py}	= yield strength of pretensioning steel	[LRFD]
f_r	= the modulus of rupture of concrete	[STD]
f_r	= modulus of rupture of concrete	[LRFD]
f_s	= allowable stress in steel	
f'_s	= ultimate stress of pretensioning reinforcement	[STD]
f_{se}	= effective final pretension stress	
f_{si}	= effective initial pretension stress	
f^*_{su}	= average stress in pretensioning steel at ultimate load	[STD]
f_t	= concrete stress at top fiber of the beam for the non-composite section	
f_{tc}	= concrete stress at top fiber of the slab for the composite section	
f_{tg}	= concrete stress at top fiber of the beam for the composite section	
f_y	= yield strength of reinforcing bars	[STD]
f_y	= specified minimum yield strength of reinforcing bars	[LRFD]
f_y	= yield stress of pretensioning reinforcement	[STD]
f'_y	= specified minimum yield strength of compression reinforcement	[LRFD]
f_{yh}	= specified yield strength of transverse reinforcement	[LRFD]
H	= average annual ambient mean relative humidity, percent	[LRFD]
H	= height of wall	[LRFD]
h	= overall depth of precast beam	[STD]
h	= overall depth of a member	[LRFD]

h_c	= total height of composite section	
h_f	= compression flange depth	[LRFD]
I	= moment of inertia about the centroid of the non-composite precast beam	[STD]
I	= moment of inertia about the centroid of the non-composite precast beam	[LRFD]
I	= impact fraction (maximum 30%)	[STD]
I_c	= moment of inertia of composite section	
IM	= dynamic load allowance	[LRFD]
J	= St. Venant torsional constant	
K	= longitudinal stiffness parameter	[STD]
K_g	= longitudinal stiffness parameter	[LRFD]
k	= factor used in calculation of distribution factor for multi-beam bridges	[LRFD]
k	= factor used in calculation of average stress in pretensioning steel for Strength Limit State	
L	= live load	[STD]
L	= length in feet of the span under consideration for positive moment and the average of two adjacent loaded spans for negative moment	[STD]
L	= overall beam length or design span	
L	= span length measured parallel to longitudinal beams	[STD]
L	= span length	[LRFD]
L_c	= critical length of yield line failure pattern	[LRFD]
LL	= vehicular live load	[LRFD]
ℓ_d	= development length	[LRFD]
ℓ_x	= length required to fully develop the strand measured from the end of the strand	
M_a	= negative moment at the end of the span being considered	
M_b	= negative moment at the end of the span being considered	
M_b	= unfactored bending moment due to barrier weight	
M_c	= flexural resistance of cantilevered wall	[LRFD]
M_{CIP}	= unfactored bending moment due to cast-in-place topping slab	
M_{const}	= unfactored bending moment due to construction load	
M_{col}	= bending moment due to horizontal collision force	
M_{cr}	= moment causing flexural cracking at section due to externally applied loads (after dead load)	[STD]
M_{cr}	= cracking moment	[LRFD]
M^*_{cr}	= cracking moment	[STD]
M_D	= unfactored bending moment due to diaphragm weight	
M_d	= bending moment at section due to unfactored dead load	
$M_{d/nc}$	= moment due to non-composite dead loads	[STD]
M_f	= unfactored bending moment due to fatigue truck per beam	
M_g	= unfactored bending moment due to beam self-weight	
M_{LL}	= unfactored bending moment due to lane load per beam	
M_{LL+I}	= unfactored bending moment due to live load + impact	
M_{LL+I}	= unfactored bending moment due to design vehicular load	
M_{LT}	= unfactored bending moment due to truck load with dynamic allowance per beam	

M_{max} = maximum factored moment at section due to externally applied loads [STD]
M_n = nominal moment strength of a section [STD]
M_n = nominal flexural resistance [LRFD]
$M_{n/dc}$ = non-composite dead load moment at the section
M_r = factored flexural resistance of a section in bending [LRFD]
M_s = maximum positive moment
M_s = unfactored bending moment due to slab and haunch weights
M_{SDL} = unfactored bending moment due to super-imposed dead loads
$M_{service}$ = total bending moment for service load combination
M_{SIP} = unfactored bending moment due to stay-in-place panel
M_u = factored bending moment at section [STD]
M_u = factored moment at a section [LRFD]
M_{ws} = unfactored bending moment due to wearing surface
M_x = bending moment at a distance (x) from the support
m = material parameter
m = stress ratio = $(f_y/0.85f'_c)$
N_b = number of beams [LRFD]
N_L = number of traffic lanes [STD]
N_u = applied factored axial force taken as positive if tensile [LRFD]
n = modular ratio between deck slab and beam materials
P = diaphragm weight concentrated at quarter points
P = load on one rear wheel of design truck (P_{15} or P_{20}) [STD]
P_c = permanent net compression force [LRFD]
P_{eff} = effective post-tensioning force
P_i = total pretensioning force immediately after transfer
P_{pe} = total pretensioning force after all losses
P_r = factored bursting resistance of pretensioned anchorage zone provided by transverse reinforcement
P_s = prestress force before initial losses
P_{se} = effective pretension force after allowing for all losses
P_{si} = effective pretension force after allowing for the initial losses
P_{20} = load on one rear wheel of the H20 truck [STD]
Q = total factored load [LRFD]
Q_i = specified loads [LRFD]
q = generalized load [LRFD]
RH = relative humidity [STD]
R_n = coefficient of resistance
R_u = flexural resistance factor
R_w = total transverse resistance of the railing or barrier [LRFD]
S = width of precast beam [STD]
S = average spacing between beams in feet [STD]
S = spacing of beams [LRFD]

S	= span length of deck slab	[STD]
S	= effective span length of the deck slab; clear span plus distance from extreme flange tip to face of web	LRFD]
S_b	= section modulus for the extreme bottom fiber of the non-composite precast beam	[STD]
S_{bc}	= composite section modulus for extreme bottom fiber of the precast beam (equivalent to S_c in the *Standard Specifications*)	
SH	= loss of pretension due to concrete shrinkage	[STD]
SR	= fatigue stress range	
S_t	= section modulus for the extreme top fiber of the non-composite precast beam	
S_{tc}	= composite section modulus for top fiber of the deck slab	
S_{tg}	= composite section modulus for top fiber of the precast beam	
s	= longitudinal spacing of the web reinforcement	[STD]
s	= length of a side element	[LRFD]
s	= spacing of rows of ties	[LRFD]
T	= collision force at deck slab level	
t	= thickness of web	
t	= thickness of an element of the beam	
t_f	= thickness of flange	
t_s	= cast-in-place deck thickness	
t_s	= depth of concrete deck	[LRFD]
V_c	= nominal shear strength provided by concrete	[STD]
V_c	= nominal shear resistance provided by tensile stresses in the concrete	[LRFD]
V_{ci}	= nominal shear strength provided by concrete when diagonal cracking results from combined shear and moment	[STD]
V_{cw}	= nominal shear strength provided by concrete when diagonal cracking results from excessive principal tensile stress in web	[STD]
V_d	= shear force at section due to unfactored dead load	[STD]
V_i	= factored shear force at section due to externally applied loads occurring simultaneously with M_{max}	[STD]
V_{LL}	= unfactored shear force due to lane load per beam	
V_{LL+I}	= unfactored shear force due to live load plus impact	
V_{LL+I}	= unfactored shear force due design vehicular live load	
V_{LT}	= unfactored shear force due to truck load with dynamic allowance per beam	
V_{mu}	= ultimate shear force occurring simultaneously with M_u	
V_n	= nominal shear resistance of the section considered	[LRFD]
V_{nh}	= nominal horizontal shear strength	[STD]
V_p	= vertical component of effective pretension force at section	[STD]
V_p	= component in the direction of the applied shear of the effective pretensioning force, positive if resisting the applied shear	[LRFD]
V_s	= nominal shear strength provided by web reinforcement	[STD]
V_s	= shear resistance provided by shear reinforcement	[LRFD]
V_u	= factored shear force at the section	[STD]

V_u = factored shear force at section [LRFD]

V_{uh} = factored horizontal shear force per unit length of the beam [LRFD]

V_x = shear force at a distance (x) from the support

v = factored shear stress [LRFD]

W = overall width of bridge measured perpendicular to the longitudinal beams [STD]

w = a uniformly distributed load [LRFD]

w = width of clear roadway [LRFD]

w_b = weight of barriers

w_c = unit weight of concrete [STD]

w_c = unit weight of concrete [LRFD]

w_g = beam self-weight

w_s = slab and haunch weights

w_{ws} = weight of future wearing surface

X = distance from load to point of support [STD]

x = the distance from the support to the section under question

y_b = distance from centroid to the extreme bottom fiber of the non-composite precast beam

y_{bc} = distance from the centroid of the composite section to extreme bottom fiber of the precast beam

y_{bs} = distance from the center of gravity of strands to the bottom fiber of the beam

y_t = distance from centroid to the extreme top fiber of the non-composite precast beam

y_{tc} = distance from the centroid of the composite section to extreme top fiber of the slab

y_{tg} = distance from the centroid of the composite section to extreme top fiber of the precast beam

Z (or z)= factor reflecting exposure conditions [LRFD], [STD]

α = angle of inclination of transverse reinforcement to longitudinal axis

β = factor indicating ability of diagonally cracked concrete to transmit tension (a value indicating concrete contribution) [LRFD]

β_D = load combination coefficient for dead loads [STD]

β_L = load combination coefficient for live loads [STD]

β_1 = factor for concrete strength [STD]

β_1 = ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength limit state to the depth of the actual compression zone [LRFD]

Δ_{beam} = deflection due to beam self-weight

Δ_{b+ws} = deflection due to barrier and wearing surface weights

Δf_{cdp} = change in concrete stress at center of gravity of pretensioning steel due to dead loads except the dead load acting at the time of the pretensioning force is applied [LRFD]

Δf_{pCR} = loss in pretensioning steel stress due to creep [LRFD]

Δf_{pES} = loss in pretensioning steel stress due to elastic shortening [LRFD]

Δf_{pi} = total loss in pretensioning steel stress immediately after transfer

Δf_{pR} = loss in pretensioning steel stress due to relaxation of steel [LRFD]

Δf_{pR1} = loss in pretensioning steel stress due to relaxation of steel at transfer [LRFD]

Δf_{pR2} = loss in pretensioning steel stress due to relaxation of steel after transfer [LRFD]

Δf_{pSR} = loss in pretensioning steel stress due to shrinkage [LRFD]

Δf_{pT}	= total loss in pretensioning steel stress	[LRFD]
Δ_D	= deflection due to diaphragm weight	
Δ_L	= deflection due to specified live load	
Δ_{LL+I}	= deflection due to live load and impact	
Δ_{LL}	= deflection due to lane load	
Δ_{LT}	= deflection due to design truck load and impact	
Δ_{max}	= maximum allowable live load deflection	
Δ_p	= camber due pretension force at transfer	
Δ_{SDL}	= deflection due to barrier and wearing surface weights	
Δ_{slab}	= deflection due to the weights of slab and haunch	
ε_x	= longitudinal strain in the web reinforcement on the flexural tension side of the member	[LRFD]
γ	= load factor	[STD]
γ^*	= factor for type of pretensioning reinforcement, 0.28 for low relaxation strand	[STD]
γ_i	= load factor	[LRFD]
η	= load modifier (a factor relating to ductility, redundancy, and operational importance)	[LRFD]
ϕ	= strength reduction factor for moment = 1.0	[STD]
ϕ	= strength reduction factor for shear = 0.90	[STD]
ϕ	= resistance factor	[LRFD]
λ	= parameter used to determine friction coefficient μ	[LRFD]
μ	= Poisson's ratio for beams	[STD]
μ	= coefficient of friction	[LRFD]
θ	= angle of inclination of diagonal compressive stresses	[LRFD]
ρ_{actual}	= actual ratio of non-pretensioned reinforcement	
ρ_b	= reinforcement ratio producing balanced strain condition	[STD]
ρ^*	$= \dfrac{A_s^*}{bd}$, ratio of pretensioning reinforcement	[STD]
ψ	= angle of harped pretensioned reinforcement	

TABLE OF CONTENTS

BULB-TEE (BT-72), SINGLE SPAN, COMPOSITE DECK, STANDARD SPECIFICATIONS

9.3.1 INTRODUCTION

9.3.2 MATERIALS

9.3.3 CROSS-SECTION PROPERTIES FOR A TYPICAL INTERIOR BEAM

9.3.3.1 Non-Composite Section

9.3.3.2 Composite Section

9.3.3.2.1 Effective Flange Width

9.3.3.2.2 Modular Ratio Between Slab and Beam Materials

9.3.3.2.3 Transformed Section Properties

9.3.4 SHEAR FORCES AND BENDING MOMENTS

9.3.4.1 Shear Forces and Bending Moments Due to Dead Loads

9.3.4.1.1 Dead Loads

9.3.4.1.2 Unfactored Shear Forces and Bending Moments

9.3.4.2 Shear Forces and Bending Moments Due to Live Load

9.3.4.2.1 Live Load

9.3.4.2.2 Live Load Distribution Factor for a Typical Interior Beam

9.3.4.2.3 Live Load Impact

9.3.4.2.4 Unfactored Shear Forces and Bending Moments

9.3.4.3 Load Combinations

9.3.5 ESTIMATE REQUIRED PRESTRESS

9.3.5.1 Service Load Stresses at Midspan

9.3.5.2 Allowable Stress Limit

9.3.5.3 Required Number of Strands

9.3.5.4 Strand Pattern

9.3.6 PRESTRESS LOSSES

9.3.6.1 Shrinkage

9.3.6.2 Elastic Shortening

9.3.6.3 Creep of Concrete

9.3.6.4 Relaxation of Pretensioning Steel

9.3.6.5 Total Losses at Transfer

9.3.6.6 Total Losses at Service Loads

TABLE OF CONTENTS

BULB-TEE (BT-72), SINGLE SPAN, COMPOSITE DECK, STANDARD SPECIFICATIONS

9.3.7 CONCRETE STRESSES AT TRANSFER

9.3.7.1 Allowable Stress Limits
9.3.7.2 Stresses at Transfer Length Section
9.3.7.3 Stresses at Harp Points
9.3.7.4 Stresses at Midspan
9.3.7.5 Hold-Down Force
9.3.7.6 Summary of Stresses at Transfer

9.3.8 CONCRETE STRESSES AT SERVICE LOADS

9.3.8.1 Allowable Stress Limits
9.3.8.2 Stresses at Midspan
9.3.8.3 Summary of Stresses at Service Loads

9.3.9 FLEXURAL STRENGTH

9.3.10 DUCTILITY LIMITS

9.3.10.1 Maximum Reinforcement
9.3.10.2 Minimum Reinforcement

9.3.11 SHEAR DESIGN

9.3.12 HORIZONTAL SHEAR DESIGN

9.3.13 PRETENSIONED ANCHORAGE ZONE

9.3.13.1 Minimum Vertical Reinforcement
9.3.13.2 Confinement Reinforcement

9.3.14 DEFLECTION AND CAMBER

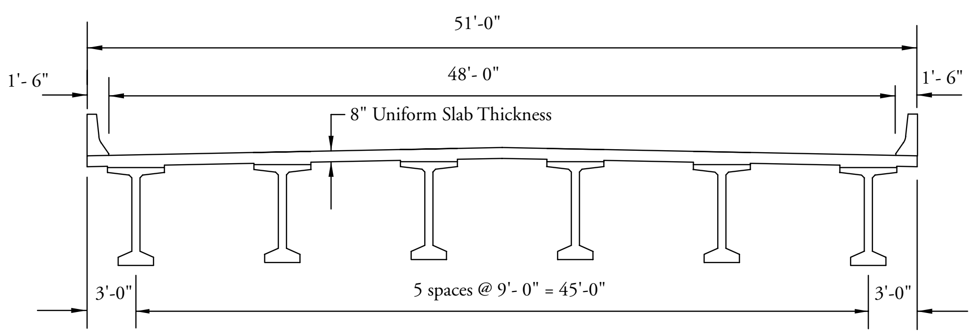
9.3.14.1 Deflection Due to Pretensioning Force at Transfer
9.3.14.2 Deflection Due to Beam Self-Weight
9.3.14.3 Total Initial Deflection
9.3.14.4 Deflection Due to Slab and Haunch Weights
9.3.14.5 Deflection Due to Barrier and Wearing Surface Weights
9.3.14.6 Deflection Due to Live Load and Impact

Bulb-Tee (BT-72), Single Span, Composite Deck, Standard Specifications

9.3.1 INTRODUCTION

This design example demonstrates the design of a 120-ft single-span AASHTO-PCI bulb-tee beam bridge. This example illustrates in detail the design of a typical interior beam at the critical sections in positive flexure, shear and deflection. The superstructure consists of six beams spaced at 9 ft on-center. Beams are designed to act compositely with the 8 in. thick cast-in-place concrete slab to resist all superimposed dead loads, live loads and impact. The top 1/2 in. of the slab is considered to be a wearing surface. Design live load is AASHTO HS20. The design is carried out in accordance with the AASHTO *Standard Specification for Highway Bridges,* 17th Edition, 2002.

Figure 9.3.1-1
Bridge Cross-section



9.3.2 MATERIALS

Cast-in-place slab: Thickness, actual, t_s = 8.0 in.

Structural = 7.5 in.

Note that a 1/2 in. wearing surface is considered to be an integral part of the 8 in. slab.

Concrete strength at 28 days, f'_c = 4,000 psi

Precast beams: AASHTO-PCI BT-72 Bulb-tee (as shown in **Fig. 9.3.2-1**)

Concrete strength at release, f'_{ci} = 5,500 psi

Concrete strength at 28 days, f'_c = 6,500 psi

Concrete unit weight = 150 pcf

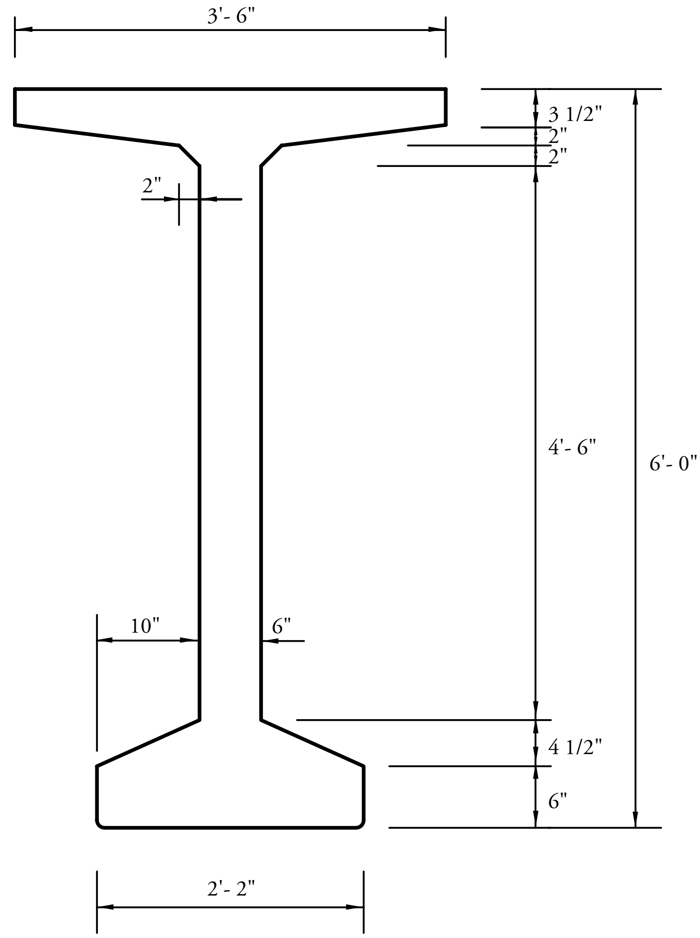
Overall beam length = 121.0 ft

Design span = 120.0 ft

BULB-TEE (BT-72), SINGLE SPAN, COMPOSITE DECK, STANDARD SPECIFICATIONS

9.3.2 Materials

Figure 9.3.2-1
AASHTO-PCI BT-72 Bulb-Tee



Pretensioning strands: 1/2 in. diameter, seven wire, low relaxation

Area of one strand = 0.153 in.^{2}

Ultimate stress, $f_s' = 270{,}000$ psi

Yield strength, $f_y^* = 0.9\ f_s' = 243{,}000$ psi [STD Art. 9.1.2]

Initial pretensioning, $f_{si} = 0.75\ f_s' = 202{,}500$ psi [STD Art. 9.15.1]

Modulus of elasticity, $E_s = 28{,}500$ ksi

Although the *Standard Specifications,* [Art. 9.16.2.1.2] indicates that the modulus of elasticity, E_s is 28,000 ksi, a value of 28,500 ksi is a more accurate value, according to the PCI Design Handbook and the *LRFD Specifications.*

Reinforcing bars: Yield strength, $f_y = 60{,}000$ psi

Future wearing surface: additional 2 in. with unit weight = 150 pcf

New Jersey-type barrier weight = 300 lbs/ft/side

9.3.3 CROSS-SECTION PROPERTIES FOR A TYPICAL INTERIOR BEAM

9.3.3.1 Non-Composite Section

A = area of cross section of precast beam = 767 in.^{2}

h = overall depth of precast beam = 72 in.

I = moment of inertia about the centroid of the non-composite precast beam = 545,894 in.^{4}

y_b = distance from centroid to the extreme bottom fiber of the non-composite precast beam = 36.60 in.

y_t = distance from centroid to the extreme top fiber of the non-composite precast beam = 35.40 in.

S_b = section modulus for the extreme bottom fiber of the non-composite precast beam = I/y_b = 14,915 in.^{3}

S_t = section modulus for the extreme top fiber of the non-composite precast beam = I/y_t = 15,421 in.^{3}

Wt. = 0.799 k/ft

E_c = modulus of elasticity of concrete, psi

where

$E_c = (w_c)^{1.5}(33)\sqrt{f_c'}$ [STD Art. 8.7.1]

w_c = unit weight of concrete = 150 pcf [STD Art. 3.3.6]

The *Standard Specifications* [STD Art. 8.7.1] indicates that the unit weight of normal weight concrete is 145 pcf. However, precast concrete mixes typically have a relatively low water/cement ratio and high density. Therefore, a unit weight of 150 pcf is used in this example. For high strength concrete, this value may need to be increased based on test results.

f_c' = specified strength of concrete, psi

Modulus of elasticity for the cast-in-place slab, using f_c' = 4,000 psi, is:

$$E_c = (150)^{1.5}(33)\sqrt{4{,}000}/1000 = 3{,}834 \text{ ksi}$$

Modulus of elasticity for the beam at release, using $f_c' = f_{ci}'$ = 5,500 psi, is:

$$E_{ci} = (150)^{1.5}(33)\sqrt{5{,}500}/1000 = 4{,}496 \text{ ksi}$$

Modulus of elasticity of the beam at service loads, using f_c' = 6,500 psi is:

$$E_c = (150)^{1.5}(33)\sqrt{6{,}500}/1000 = 4{,}888 \text{ ksi}$$

9.3.3.2
Composite Section

9.3.3.2.1
Effective Flange Width

[STD Art. 9.8.3]

Effective web width of the precast beam is the lesser of: [STD Art. 9.8.3.1]

b_e = top flange width = 42 in. (controls)

or, $b_e = 2(6)(5.5) + 6 + 2(2) = 76$ in.

Effective web width, $b_e = 42$ in.

The effective flange width is the lesser of: [STD Art. 9.8.3.2]

1/4 span length: $\dfrac{120(12)}{4} = 360$ in.

Distance center-to-center of beams: 9(12) = 108 in. (controls)

12 (effective slab thickness) plus effective beam web width

12(7.5) + 42 = 132 in.

Effective flange width = 108 in.

9.3.3.2.2
Modular Ratio Between Slab and Beam Materials

Modular ratio between slab and beam materials:

$$n = \frac{E_c(\text{slab})}{E_c(\text{beam})} = \frac{3{,}834}{4{,}888} = 0.7845$$

9.3.3.2.3
Transformed Section Properties

Note: Only the structural thickness of the deck (7.5 in.) will be used in these computations.

Transformed flange width = n (effective flange width) = 0.7845 (108) = 84.73 in.

Transformed flange area = n (effective flange width)(t_s) = 0.7845(108)(7.5) = 635.45 in^2

Due to camber of the pretensioned precast beam, a minimum haunch thickness of 1/2 in. at midspan is considered in the structural properties of the composite section. The haunch width must also be transformed.

Transformed haunch width = (0.7845)(42) = 32.95 in.

Transformed area of haunch = (0.7845)(42)(0.5) = 16.47 in.^{2}

Note that the haunch should only be considered to contribute to section properties if it is required to be provided in the completed structure. Therefore, some designers neglect its contribution to the section properties.

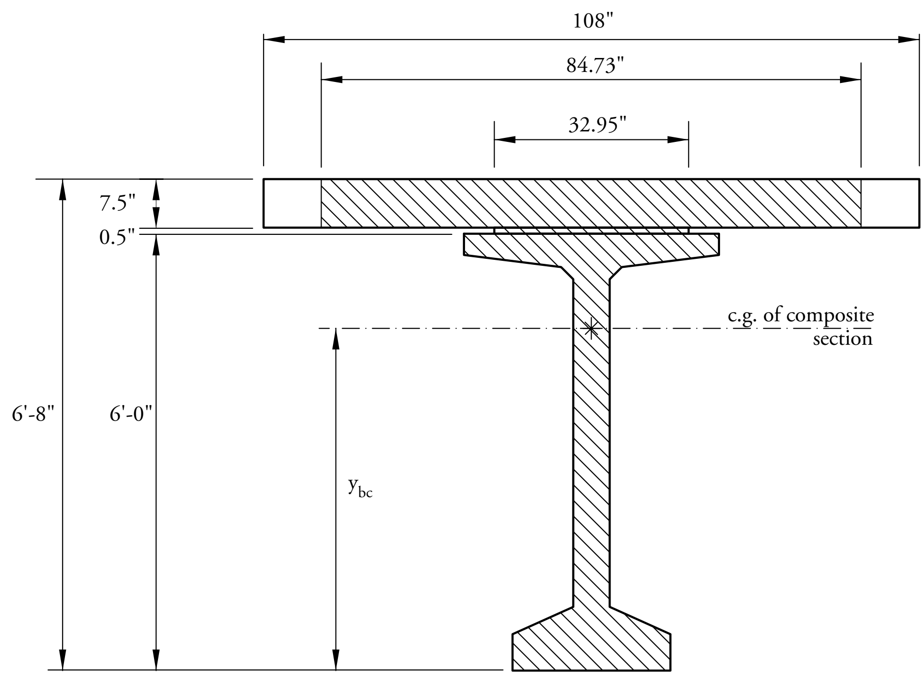
BULB-TEE (BT-72), SINGLE SPAN, COMPOSITE DECK, STANDARD SPECIFICATIONS

9.3.3.2.3 Transformed Section Properties

Table 9.3.3.2.2-1
Properties of Composite Section

	Transformed Area in.^{2}	y_b in.	A y_b in.^{3}	A $(y_{bc}-y_b)^2$	I in.^{4}	I+ A $(y_{bc}-y_b)^2$ in.^{4}
Beam	767.00	36.60	28,072.20	253,224.21	545,894.00	799,118.21
1/2" Haunch	16.47	72.25	1,189.96	5,032.42	0.34	5,032.76
Slab	635.45	76.25	48,453.06	293,190.53	2,978.79	296,169.32
Σ	1,418.92		77,715.22			1,100,320.20

Figure 9.3.3.2.3-1
Composite Section



A_c = total area of composite section = 1,419 in.^{2}

h_c = total height of composite section = 80.00 in.

I_c = moment of inertia of composite section = 1,100,320 in.^{4}

y_{bc} = distance from the centroid of the composite section to extreme bottom fiber of the precast beam = 77,715/1,419 = 54.77 in.

y_{tg} = distance from the centroid of the composite section to extreme top fiber of the precast beam = 72 − 54.77 = 17.23 in.

y_{tc} = distance from the centroid of the composite section to extreme top fiber of the slab = 80 − 54.77 = 25.23 in.

S_{bc} = composite section modulus for extreme bottom fiber of the precast beam

$$= I_c/y_{bc} = \frac{1{,}100{,}320}{54.77} = 20{,}090 \text{ in.}^3$$

S_{tg} = composite section modulus for top fiber of the precast beam

$$= I_c/y_{tg} = \frac{1{,}100{,}320}{17.23} = 63{,}861 \text{ in.}^3$$

S_{tc} = composite section modulus for top fiber of the slab

$$= \frac{I_c}{n\ y_{tc}} = \frac{1{,}100{,}320}{0.7845\,(25.23)} = 55{,}592 \text{ in.}^3$$

9.3.4 SHEAR FORCES AND BENDING MOMENTS

The self-weight of the beam and the weight of the slab and haunch act on the non-composite simple-span structure, while the weight of barriers, future wearing surface, and live load plus impact act on the composite simple-span structure.

9.3.4.1 Shear Forces and Bending Moments Due to Dead Loads

[STD Art. 3.3]

9.3.4.1.1 Dead Loads

Beam weight = 0.799 kip/ft

$$\text{8 in. slab weight} = 0.150\left(\frac{8}{12}\right)(9.0) = 0.900 \text{ kip/ft}$$

$$\text{Haunch weight} = \frac{1/2}{12}(3.5 \text{ ft})(0.150) = 0.022 \text{ kip/ft}$$

Note:

1. Actual slab thickness (8 in.) is used for computing slab dead load
2. A 1/2 in. minimum haunch is assumed in the computation of forces. If a deeper haunch will be used because of final beam camber, the weight of the actual haunch should be used.
3. Diaphragms: Many state agencies are moving away from using cast-in-place concrete diaphragms in favor of lighter weight steel diaphragms. Therefore, the weight of diaphragms will be ignored.
4. Dead loads placed on the composite structure are distributed equally among all beams. [STD Art. 3.23.2.3.1.1]

$$\text{Barriers: (2 barriers)}\ \frac{300}{6 \text{ beams}}/1{,}000 = 0.100 \text{ kip/ft/beam}$$

Weight of future wearing surface: $\frac{2}{12}(150) = 25$ psf which is applied over the entire width of the bridge between curbs (48 ft)

$$\text{Future wearing surface: } \frac{25(48.0)}{6}/1{,}000 = 0.200 \text{ kip/ft/beam}$$

9.3.4.1.2
Unfactored Shear Forces and Bending Moments

For a simply supported beam, simple span (L), loaded with a uniformly distributed load (w), the shear force (V_x) and the bending moment (M_x) at a distance (x) from the support are given by:

$$V_x = w(0.5L - x) \qquad \text{(Eq. 9.3.4.1.2-1)}$$

$$M_x = 0.5wx(L - x) \qquad \text{(Eq. 9.3.4.1.2-2)}$$

Using the above equations, values of shear forces and bending moments for a typical interior beam under dead loads (weight of beam, slab, haunch, barriers and future wearing surface) are computed and given in **Table 9.3.4.1.2-1**. For these calculations, the span length is the design span (120 ft). However, for calculations of stresses and deformations at the time the prestress is released, the overall length of the precast beam (121 ft) is used, as illustrated later in this example.

9.3.4.2
Shear Forces and Bending Moments Due to Live Load

9.3.4.2.1
Live Load

Live load is either the standard truck or lane loading corresponding to HS20. The standard truck load will govern the design for this 120-ft simple-span example.

[STD Art. 3.7.1.1]

9.3.4.2.2
Live Load Distribution Factor for a Typical Interior Beam

Using the live load distribution factor for moment for a precast pretensioned concrete beam, the fraction of the wheel load carried by the interior beam:

$$DF_m = \frac{S}{5.5} = \frac{9.0}{5.5} = 1.636 \text{ wheels/beam} \qquad \text{[STD Table 3.23.1]}$$

where S = average spacing between beams in feet

$DF_m/2 = 0.818$ lanes/beam

[STD Art. 3.8]

9.3.4.2.3
Live Load Impact

The live load impact factor is computed using in the following equation:

$$I = \frac{50}{L + 125} \qquad \text{[STD Eq. 3-1]}$$

where

I = impact fraction (maximum 30%)

L = length in feet of the span under consideration = 120 ft [STD Art. 3.8.2.2]

$$I = \frac{50}{120 + 125} = 0.204$$

Impact for shear varies along the span according to the location of the truck [STD Art. 3.8.2.2 (d)]. For simplicity, the impact factor computed above is used for shear.

9.3.4.2.4
Unfactored Shear Forces and Bending Moments

Shear force and bending moment envelopes on a per-lane-basis are calculated at tenth-points of the span using the equations given in Chapter 8. However, this generally is done by means of commercially available computer software that has the ability to deal with moving loads.

Therefore, live load shear force and bending moment per beam are:

V_{LL+I} = (shear force per lane)(Distribution Factor)(1+I)

= (shear force per lane)(0.818)(1+0.204)

= (shear force per lane)(0.985), kips

M_{LL+I} = (bending moment per lane)(Distribution Factor)(1+I)

= (bending moment per lane)(0.818)(1+0.204)

= (bending moment per lane)(0.985), ft-kips

At any section along the span, the maximum bending moment and shear are computed for the standard truck loading and for the lane loading separately. The larger of the two loading types controls the design for the section in question. At each section, the load position must be determined to give the maximum shears and moments. This can be done by means of commercially available software programs.

Values of V_{LL+I} and M_{LL+I} at different points are given in **Table 9.3.4.1.2-1.**

9.3.4.3 Load Combinations

[STD Art. 3.22]

For service load design (Group I): 1.00 D + 1.00(L+I) [STD Table 3.22.1A]

where

D = dead load

L = live load

I = impact fraction

For load factor design (Group I): 1.3[1.00 D + 1.67(L + I)] [STD Table 3.22.1A]

9.3.5 ESTIMATE REQUIRED PRESTRESS

The required number of strands is usually governed by concrete tensile stresses at the bottom fiber at the section of maximum moment or at the harp points. For estimating the number of strands, only the stresses at midspan need to be considered.

9.3.5.1 Service Load Stresses at Midspan

Bottom tensile stresses due to applied loads:

$$f_b = \frac{M_g + M_s}{S_b} + \frac{M_b + M_{ws} + M_{LL+I}}{S_{bc}}$$

where

f_b = concrete stress at the bottom fiber of the beam

M_g = unfactored bending moment due to beam self-weight, ft-kips

M_s = unfactored bending moment due to slab and haunch weights, ft-kips

M_b = unfactored bending moment due to barrier weight, ft-kips

M_{ws} = unfactored bending moment due to wearing surface, ft-kips

M_{LL+I} = unfactored bending moment due to live load + impact, ft-kips

Using values of bending moments from **Table 9.3.4.1.2-1**, the bottom tensile stress at midspan is:

$$f_b = \frac{(1,438.2+1,659.6)(12)}{14,915} + \frac{(180.0+360.0+1,851.6)(12)}{20,090} = 3.921 \text{ ksi}$$

Table 9.3.4.1.2-1 Unfactored Shear Forces and Bending Moments for a Typical Interior Beam

Distance	Section	Beam weight		Slab + Haunch weights		Barrier weight		Wearing Surface weight		Live load + impact	
x	x/L	Shear V_g	Moment M_g	Shear V_s	Moment M_s	Shear V_b	Moment M_b	Shear V_{ws}	Moment M_{ws}	Shear V_{LL+I}	Moment M_{LL+I}
ft		kips	ft-kips	kips	ft-kips	kips	ft-kips	kips	ft-kips	kips	ft-kips
0	0.000	47.9	0.0	55.3	0.0	6.0	0.0	12.0	0.0	65.4	0.0
3.333[1]	0.028	45.3	155.4	52.2	179.3	5.7	19.4	11.3	38.9	63.6	211.5
12	0.100	38.4	517.8	44.3	597.5	4.8	64.8	9.6	129.6	58.3	699.7
24	0.200	28.8	920.4	33.2	1,062.1	3.6	115.2	7.2	230.4	51.2	1,229.1
36	0.300	19.2	1,208.1	22.1	1,394.1	2.4	151.2	4.8	302.4	44.1	1,588.4
48[2]	0.400	9.6	1,380.7	11.1	1,593.2	1.2	172.8	2.4	345.6	37.0	1,799.6
60	0.500	0.0	1,438.2	0.0	1,659.6	0.0	180.0	0.0	360.0	29.9	1,851.6

[1] Critical section for shear (see section 9.3.11 of this example)
[2] Harp point

9.3.5.2 Allowable Stress Limit

At service loads, allowable tensile stress in the precompressed tensile zone:

$$F_b = 6\sqrt{f_c'} = 6\sqrt{6{,}500}\left(\frac{1}{1{,}000}\right) = -0.484\text{ksi} \qquad \text{[STD Art. 9.15.2.2]}$$

9.3.5.3 Required Number of Strands

Required precompressive stress in the bottom fiber after losses:

Bottom tensile stress - allowable tension stress at final = $f_b - F_b$

$= 3.921 - 0.484 = 3.437$ ksi

The location of the center of gravity of strands at midspan usually ranges from 5 to 15% of the beam depth, measured from the bottom of the beam. A value of 5% is appropriate for newer efficient sections like bulb-tees and 15% for the less efficient AASHTO shapes of normal design. Assume the distance from the center of gravity of strands to the bottom fiber of the beam is equal to y_{bs} = 4.00 in.
(Note: $y_{bs}/h = 4/72 = 5.5\%$)

Strand eccentricity at midspan:

$e_c = y_b - y_{bs} = 36.60 - 4.00 = 32.60$ in.

Bottom fiber stress due to prestress after all losses:

$$f_b = \frac{P_{se}}{A} + \frac{P_{se}\, e_c}{S_b}$$

where P_{se} = effective pretension force after allowing for all losses

Set the required precompression (3.437 ksi) equal to the bottom fiber stress due to prestress, solve for the required minimum P_{se}.

$$\text{Then, } 3.437 = \frac{P_{se}}{767} + \frac{32.60\, P_{se}}{14{,}915}$$

P_{se} = 985.0 kips

Assume final losses = 25% of f_{si}

Assumed final losses = 0.25(202.5) = 50.6 ksi

The available prestress force per strand after all losses

= (cross-sectional area of one strand)$[f_{si}$ – losses]

$= 0.153(202.5 - 50.6) = 23.24$ kips

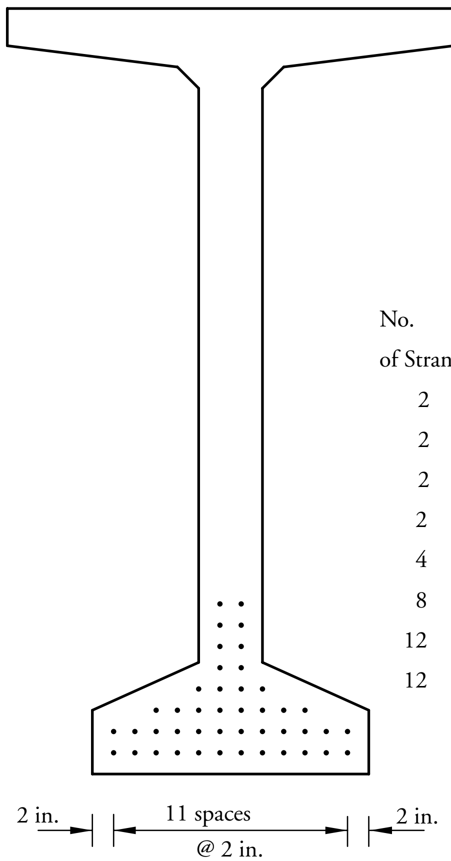
Number of strands required $= \dfrac{985.0}{23.24} = 42.38$

Try (44) 1/2-in.-diameter, 270 ksi strands

9.3.5.4 Strand Pattern

The assumed strand pattern for the 44 strands at the midspan section is shown in **Figure 9.3.5.4-1**. Each available position was filled beginning with the bottom row.

Figure 9.3.5.4-1 Assumed Strand Pattern at Midspan



No. of Strands	Distance from bottom (in.)
2	16
2	14
2	12
2	10
4	8
8	6
12	4
12	2

Calculate the distance from center of gravity of the strand to the bottom fiber of the beam, y_{bs}.

$$y_{bs} = \frac{12(2)+12(4)+8(6)+4(8)+2(10)+2(12)+2(14)+2(16)}{44} = 5.82 \text{ in.}$$

Strand eccentricity at midspan:

$e_c = y_b - y_{bs} = 36.60 - 5.82 = 30.78$ in.

9.3.6 PRESTRESS LOSSES

[STD Art. 9.16.2]

Total losses = SH + ES + CR_c + CR_s [STD Eq. 9-3]

where

SH = loss of prestress due to concrete shrinkage, ksi

ES = loss of prestress due to elastic shortening, ksi

Cr_c = loss of prestress due to creep of concrete, ksi

Cr_s = loss of prestress due to relaxation of pretensioning steel, ksi

9.3.6.1 Shrinkage

[STD Art. 9.16.2.1.1]

Relative humidity varies significantly from one area of the country to another. Refer to the U.S. map, Figure 9.16.7.1.1, in the *Standard Specifications.*

Assume relative humidity, RH = 70%

$$SH = 17{,}000 - 150\,RH = [17{,}000 - 150(70)]\frac{1}{1{,}000} = 6.5 \text{ ksi} \qquad \text{[STD Eq. 9-4]}$$

9.3.6.2 Elastic Shortening

[STD Art. 9.16.2.1.2]

For pretensioned members

$$ES = \frac{E_s}{E_{ci}} f_{cir} \qquad \text{[STD Eq. 9-6]}$$

where

$$f_{cir} = \frac{P_{si}}{A} + \frac{P_{si}e_c^{\,2}}{I} - \frac{(M_g + M_D)e_c}{I}$$

f_{cir} = average concrete stress at the center of gravity of the pretensioning steel due to pretensioning force and dead load of beam immediately after transfer

P_{si} = pretension force after allowing for the initial losses. *The Standard Specifications* allow that the reduction to initial tendon stress be estimated as 0.69 f_s' for low-relaxation strands

= (number of strands)(area of strands)(0.69f_s')

= (44)(0.153)[(0.69)(270)] = 1,254.2 kips

M_g = unfactored bending moment due to beam self-weight = 1,438.2 ft-kips

M_D = unfactored bending moment due to diaphragm weight

e_c = eccentricity of the strand at the midspan = 30.78 in.

M_g should be calculated based on the overall beam length of 121 ft. However, since the elastic shortening losses will be a part of the total losses, f_{cir} is conservatively computed based on using the design span length of 120 ft.

$$f_{cir} = \frac{1{,}254.2}{767} + \frac{1{,}254.2(30.78)^2}{545{,}894} - \frac{1{,}438.2(12)(30.78)}{545{,}894}$$

$$= 1.635 + 2.177 - 0.973 = 2.839 \text{ ksi}$$

$$ES = \frac{28{,}500}{4{,}496}(2.839) = 18.0 \text{ ksi}$$

9.3.6.3
Creep of Concrete

[STD Art. 9.16.2.1.3]

$CR_c = 12f_{cir} - 7f_{cds}$ [STD Eq. 9-9]

where

f_{cds} = concrete stress at the center of gravity of the pretensioning steel due to all dead loads except the dead load present at the time the pretensioning force is applied.

$$= \frac{M_s e_c}{I} + \frac{M_{SDL}(y_{bc} - y_{bs})}{I_c}$$

where

M_s = slab and haunch moment = 1,659.6 ft-kips

M_{SDL} = superimposed dead load moment = $M_b + M_{ws}$ = 540.0 ft-kips

y_{bc} = 54.77 in.

y_{bs} = the distance from center of gravity of the strand at midspan to the bottom of the beam = 5.82 in.

I = moment of inertia for the non-composite section = 545,894 in.^{4}

I_c = moment of inertia for the composite section = 1,100,320 in.^{4}

$$f_{cds} = \frac{1{,}659.6(12)(30.78)}{545{,}894} + \frac{(540)(12)(54.77 - 5.82)}{1{,}100{,}320}$$

= 1.123 + 0.288 = 1.411 ksi

CR_c = 12(2.839) − 7(1.411) = 24.2 ksi

9.3.6.4
Relaxation of Pretensioning Steel

[STD Art. 9.16.2.1.4]

For pretensioned members with 270 ksi low-relaxation strand:

$CR_s = 5{,}000 - 0.10\ ES - 0.05(SH + CR_c)$ [STD Eq. 9-10A]

$$= [5{,}000 - 0.10(17{,}996) - 0.05(6{,}500 + 24{,}200)]\left(\frac{1}{1{,}000}\right) = 1.7 \text{ ksi}$$

9.3.6.5
Total Losses at Transfer

Total initial losses = 18.0 ksi

f_{si} = effective initial pretension stress = 202.5 − 18.0 = 184.5 ksi

P_{si} = effective pretension force after allowing for the initial losses

= 44(0.153)(184.5) = 1,242.1 kips

9.3.6.6
Total Losses at Service Loads

SH = 6.5 ksi

ES = 18.0 ksi

CR_c = 24.2 ksi

CR_s = 1.7 ksi

Total final losses = 6.5 + 18.0 + 24.2 + 1.7 = 50.4 ksi

or $\dfrac{50.4}{0.75(270)}(100)$ = 24.9% losses

f_{se} = effective final prestress = 0.75(270) − 50.4 = 152.1 ksi

P_{se} = 44 (0.153)(152.1) = 1,023.9 kips

9.3.7 CONCRETE STRESSES AT TRANSFER

9.3.7.1 Allowable Stress Limits

[STD Art. 9.15.2.1]

Compression: $0.6\,f'_{ci} = +3.300$ ksi (compression)

Tension: The maximum tensile stress shall not exceed:

- $7.5\sqrt{f'_{ci}} = -0.556$ ksi (tension)
- If the calculated tensile stress exceeds 200 psi or $3\sqrt{f'_{ci}} = 0.222$ ksi, whichever is smaller, bonded reinforcement should be provided to resist the total tension force in the concrete computed on the assumption of an uncracked section.

9.3.7.2 Stresses at Transfer Length Section

This section is located at a distance equal to the transfer length from the end of the beam. Stresses at this location need only be checked at release, because it almost always governs. Also, losses with time will reduce the concrete stresses making them less critical.

Transfer length = 50(strand diameter) [STD Art. 9.20.2.4]

= 50(0.50) = 25 in. = 2.08 ft

The transfer length section is located at 2.08 ft from the end of the beam or at a point 1.58 ft from centerline of the bearing. This is assuming the beam extends 6 in. beyond the bearing centerline. This point on the design span = 1.58/120.0 = 0.013.

Due to the camber of the beam at release, the beam self-weight is acting on the overall beam length (121.0 ft). Therefore, the values of bending moment given in **Table 9.3.4.1.2-1** cannot be used because they are based on the design span (120.0 ft). The value of bending moment at the transfer length due to beam self-weight is calculated using Equation (9.3.4.1.2-2) based on overall length.

Therefore, $M_g = 0.5(0.799)(2.08)(121 - 2.08) = 98.8$ ft-kips.

Compute concrete stress at the top fiber of the beam, f_t:

$$f_t = +\frac{P_{si}}{A} - \frac{P_{si}\,e}{S_t} + \frac{M_g}{S_t}$$

$$f_t = +\frac{1,242.1}{767} - \frac{1,242.1(30.78)}{15,421} + \frac{98.8(12)}{15,421}$$

$$= +1.619 - 2.479 + 0.077 = -0.783 \text{ ksi}$$

Allowable tension: –0.556 ksi N.G.

Compute concrete stress at the bottom fiber of the beam, f_b:

$$f_b = +\frac{P_{si}}{A} + \frac{P_{si}\,e}{S_b} - \frac{M_g}{S_b}$$

$$f_b = +\frac{1,242.1}{767} + \frac{1,242.1(30.78)}{14,915} - \frac{98.8(12)}{14,915}$$

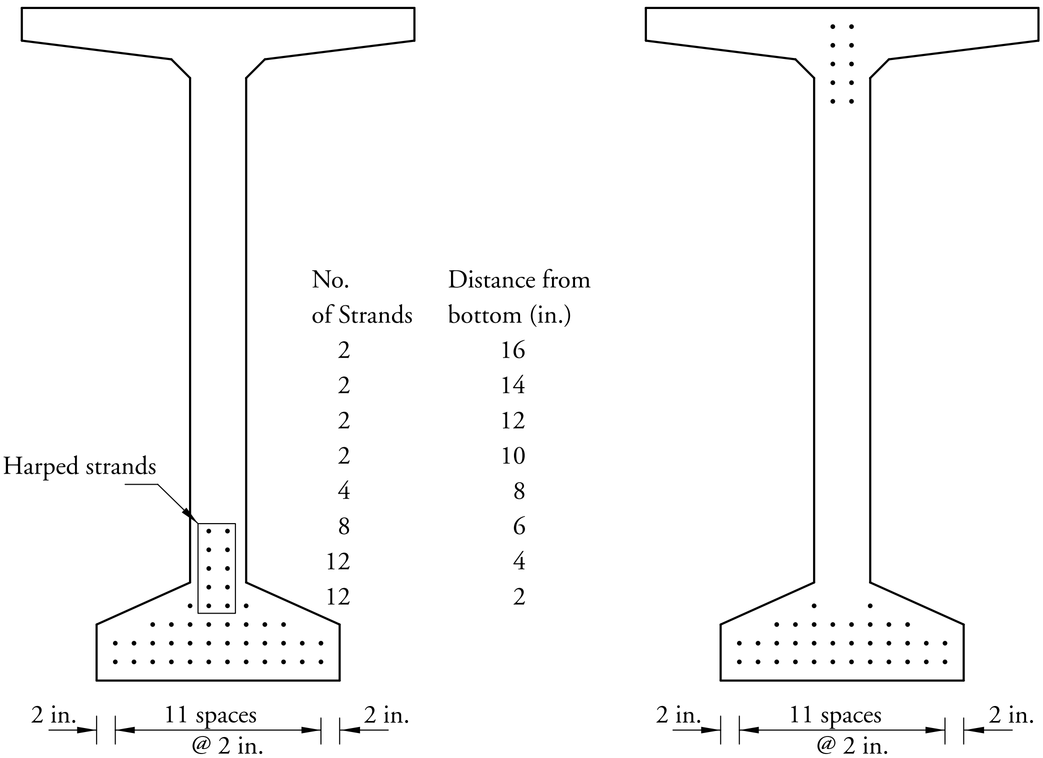
$$= +1.619 + 2.563 - 0.079 = +4.103 \text{ ksi}$$

Allowable compression: +3.300 ksi N.G.

Since the top and bottom fiber stresses exceed those allowed, harped and/or debonded strands must be used.

In this example, a harped strand pattern will be used with harp points at 0.40 L = 48.0 ft from the centerline bearing or 48.5 ft from the end of the beam. Try harping 10 strands as shown in **Figs. 9.3.7.2-1** and **9.3.7.2-2**.

Figure 9.3.7.2-1 Strand Pattern



At midspan:

No. of Strands	Distance from bottom (in.)
2	16
2	14
2	12
2	10
4	8
8	6
12	4
12	2

At ends:

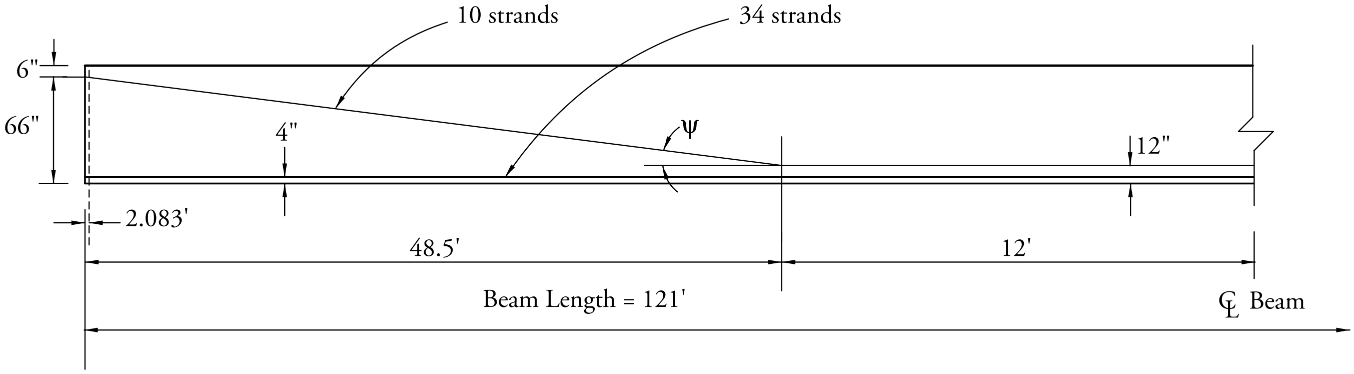
No. of Strands	Distance from bottom (in.)
2	70
2	68
2	66
2	64
2	62

No. of Strands	Distance from bottom (in.)
2	8
8	6
12	4
12	2

The distance between the center of gravity of the 10 harped strands and the top of the beam at the end of the beam $= \dfrac{2(2)+2(4)+2(6)+2(8)+2(10)}{10} = 6.00$ in.

The distance between the center of gravity of the 10 harped strands and the bottom of the beam at the harp points $= \dfrac{2(8)+2(10)+2(12)+2(14)+2(16)}{10} = 12.00$.

Figure 9.3.7.2-2 Longitudinal Strand Profile



The distance between the center of gravity of the 10 harped strands and the top of the beam at the transfer length section = $6\text{in.} + \dfrac{(72\text{in.} - 12\text{in.} - 6\text{in.})}{48.5\text{ft}}(2.083\text{ft})$
= 8.32 in.

The distance between the center of gravity of the 34 straight strands and the bottom of the beam at all locations = $\dfrac{12(2) + 12(4) + 8(6) + 2(8)}{34}$ = 4.00 in.

The distance between the center of gravity of all strands and the bottom of the beam:

at end of beam = $\dfrac{34(4) + 10(72 - 6)}{44}$ = 18.09 in.

at transfer length = $\dfrac{34(4) + 10(72 - 8.32)}{44}$ = 17.56 in.

Eccentricity of strands at transfer length, e = 36.60 − 17.56 = 19.04 in.

Recompute the concrete stresses at transfer length section with the harped strands:

$$f_t = +\frac{1,242.1}{767} - \frac{1,242.1(19.04)}{15,421} + \frac{98.8(12)}{15,421} = +1.619 - 1.534 + 0.077 = +0.162 \text{ ksi}$$

Allowable compression: +3.300 ksi O.K.

Note: Since the top fiber stress is smaller than $3\sqrt{f'_{ci}}$, there is no need for additional bonded reinforcement.

$$f_b = +\frac{1,242.1}{767} + \frac{1,242.1(19.04)}{14,915} - \frac{98.8(12)}{14,915} = +1.619 + 1.586 - 0.079 = +3.126 \text{ ksi}$$

Allowable compression: = +3.300 ksi O.K.

9.3.7.3 Stresses at Harp Points

Eccentricity of strands is the same as at midspan.

Bending moment at the harp point (0.4 L) due to beam self-weight is calculated using Equation (9.3.4.1.2-2) based on overall length.

Therefore, $M_g = 0.5(0.799)(48.5)(121 - 48.5) = 1{,}404.7$ ft-kips

$$f_t = +\frac{1,242.1}{767} - \frac{1,242.1(30.78)}{15,421} + \frac{1,404.7(12)}{15,421} = +1.619 - 2.479 + 1.093$$

$= +0.233$ ksi

Allowable compression: = +3.300 ksi O.K.

$$f_b = +\frac{1,242.1}{767} + \frac{1,242.1(30.78)}{14,915} - \frac{1,404.7(12)}{14,915} = +1.619 + 2.563 - 1.130$$

$= +3.052$ ksi

Allowable compression: +3.300 ksi O.K.

9.3.7.4 Stresses at Midspan

Bending moment at midspan due to beam self-weight is calculated using Equation (9.3.4.1.2-2) based on overall length.

Therefore, $M_g = 0.5\ (0.799)(60.5)(121 - 60.5) = 1{,}462.3$ ft-kips

$$f_t = +\frac{1,242.1}{767} - \frac{1,242.1(30.78)}{15,421} + \frac{1,462.3(12)}{15,421} = +1.619 - 2.479 + 1.138$$

$= +0.278$ ksi

Allowable compression: +3.300 ksi O.K.

$$f_b = +\frac{1,242.1}{767} + \frac{1,242.1(30.78)}{14,915} - \frac{1,462.3(12)}{14,915} = +1.619 + 2.563 - 1.177$$

$= +3.005$ ksi

Allowable compression: = +3.300 ksi O.K.

Note: Stresses at harp points are more critical than stresses at midspan.

9.3.7.5 Hold-Down Force

Assume the maximum initial pretensioning stress before allowing for any losses:

$0.80f_{pu} = 0.80(270) = 216$ ksi

Initial pretension force per strand before losses = 0.153(216) = 33.0 kips

$$\text{Hold-down force per strand} = \frac{54}{48.5(12)}(33.0\ \text{kips/strand})(1.05) = 3.21\ \text{kips/strand}$$

Note: The factor, 1.05, is applied to account for friction.

Total force = 10 strands (3.21) = 32.1 kips

$$\psi = \tan^{-1}\left(\frac{54}{48.5(12)}\right) = 5.30°$$

Where 54 in. is the vertical strand shift in a horizontal distance of 48.5 ft.

The hold-down force and the harp angle should be checked against maximum limits for local practices. Refer to Chapter 3, Fabrication and Construction, Section 3.3.2.2 and Chapter 8, Design Theory and Procedures for additional details.

9.3.7.6 Summary of Stresses at Transfer

	Top of beam f_t (ksi)	Bottom of beam f_b (ksi)
At transfer length section	+0.162	+3.126
At harp points	+0.233	+3.052
At midspan	+0.278	+3.005

9.3.8 CONCRETE STRESSES AT SERVICE LOADS

9.3.8.1 Allowable Stress Limits

[STD Art. 9.15.2.2]

Compression:

Case (I): for all load combinations

$0.60f_c' = 0.60(6{,}500)/1{,}000 = +3.900$ ksi (for precast beam)

$0.60f_c' = 0.60(4{,}000)/1{,}000 = +2.400$ ksi (for slab)

Case (II): for effective pretension force + permanent dead loads

$0.40f_c' = 0.40(6{,}500)/1{,}000 = +2.600$ ksi

$0.40f_c' = 0.40(4{,}000)/1{,}000 = +1.600$ ksi (for slab)

Case (III): live load + 1/2 (pretensioning force + dead loads)

$0.40f_c' = 0.40(6{,}500)/1{,}000 = +2.600$ ksi (for precast beam)

$0.40f_c' = 0.40(4{,}000)/1{,}000 = +1.600$ ksi (for slab)

Tension: $6\sqrt{f_c'} = 6\sqrt{6{,}500}\left(\dfrac{1}{1{,}000}\right) = -0.484$ ksi

9.3.8.2 Stresses at Midspan

Bending moment values at this section are given in **Table 9.3.4.1.2-1**.

Compute concrete stress at top fiber of beam:

$$f_t = +\frac{P_{se}}{A} - \frac{P_{se}e_c}{S_t} + \frac{M_g + M_s}{S_t} + \frac{M_b + M_{ws}}{S_{tg}} + \frac{M_{LL+I}}{S_{tg}}$$

Case (I):

$$f_t = \frac{1{,}023.9}{767} - \frac{1{,}023.9(30.78)}{15{,}421} + \frac{(1{,}438.2 + 1{,}659.6)(12)}{15{,}421} + \frac{(180 + 360)(12)}{63{,}861} + \frac{1{,}851.6(12)}{63{,}861}$$

$= +1.335 - 2.044 + 2.411 + 0.101 + 0.348 = +2.151$ ksi

Allowable compression: +3.900 ksi O.K.

Case (II):

$f_t = +1.335 - 2.044 + 2.411 + 0.101 = +1.803$ ksi

Allowable compression: +2.600 ksi O.K.

Case (III):

$f_t = 0.5(+1.335 - 2.044 + 2.411 + 0.101) + 0.348 = +1.250$ ksi

Allowable compression: +2.600 ksi O.K.

Compute concrete stresses at bottom of beam:

$$f_b = +\frac{P_{se}}{A} + \frac{P_{se}e_c}{S_b} - \frac{M_g + M_s}{S_b} - \frac{M_b + M_{ws}}{S_{bc}} - \frac{M_{LL+I}}{S_{bc}}$$

$$f_b = +\frac{1{,}023.9}{767} + \frac{1{,}023.9(30.78)}{14{,}915} - \frac{(1{,}438.2 + 1{,}659.6)(12)}{14{,}915} - \frac{(180 + 360)(12)}{20{,}090} - \frac{1{,}851.6(12)}{20{,}090}$$

$$= +1.335 + 2.113 - 2.492 - 0.323 - 1.106 = -0.473 \text{ ksi}$$

Allowable tension: −0.484 ksi O.K.

Compute stresses at the top of the slab:

Case (I):

$$f_t = \frac{M_b + M_{ws}}{S_{tc}} + \frac{M_{LL+I}}{S_{tc}} = \frac{(180 + 360)(12)}{55{,}592} + \frac{(1{,}851.6)(12)}{55{,}592} = 0.117 + 0.400$$

$$= 0.517 \text{ ksi}$$

Allowable compression: +2.400 ksi O.K.

Case (II):

$f_t = +0.117$ ksi

Allowable compression: +1.600 ksi O.K.

Case (III):

$f_t = 0.5(+0.117) + 0.400 = +0.459$ ksi

Allowable compression: +1.600 ksi O.K.

9.3.8.3 Summary of Stresses at Service Loads

	Top of Slab f_t (ksi)	Top of Beam f_t (ksi)	Bottom of Beam f_t (ksi)
At midspan	+0.517	+2.151	−0.473

9.3.9 FLEXURAL STRENGTH

[STD Art. 9.17]

Using Group I load factor design loading combination, given earlier in Section 9.3.4.3 of the *Standard Specifications:*

$$M_u = 1.3[M_g + M_s + M_b + M_{ws} + 1.67(M_{LL+I})]$$ [STD Table 3.22.1.A]

$$= 1.3[1{,}438.2 + 1{,}659.6 + 180.0 + 360.0 + 1.67(1{,}851.6)] = 8{,}749 \text{ ft-kips}$$

Compute average stress in pretensioning steel at ultimate load, f_{su}^*:

$$f_{su}^* = f_s'\left(1 - \frac{\gamma^*}{\beta_1}\rho^*\frac{f_s'}{f_c'}\right)$$ [STD Eq. 9-17]

where

f^*_{su} = average stress in pretensioning steel at ultimate load.

γ^* = 0.28 for low-relaxation strand [STD Art. 9.1.2]

$$\beta_1 = 0.85 - 0.05\frac{(f'_c - 4{,}000)}{1{,}000} \geq 0.65 \text{ when } f'_c > 4{,}000 \text{ psi}$$ [STD Art. 8.16.2.7]

$$= 0.85 - 0.05\frac{(4{,}000 - 4{,}000)}{1{,}000} = 0.85$$

$$\rho^* = \frac{A^*_s}{bd}$$

where

A^*_s = area of pretensioned reinforcement = 44(0.153) = 6.732 in.^{2}

b = effective flange width = 108 in.

y_{bs} = distance from center of gravity of the strands to the bottom fiber of the beam = 5.82 in.

d = distance from top of slab to centroid of pretensioning strands

= beam depth (h) + haunch + slab thickness − y_{bs}

= 72 + 0.5 + 7.5 − 5.82 = 74.18 in.

$$\rho^* = \frac{6.732}{108\,(74.18)} = 0.000840$$

$$f^*_{su} = 270\left[1-\left(\frac{0.28}{0.85}\right)(0.000840)\left(\frac{270}{4}\right)\right] = 265.0 \text{ ksi}$$

Compute the depth of the compression block:

$$a = \frac{A^*_s f^*_{su}}{0.85\, f'_c\, b} = \frac{6.732(265.0)}{(0.85)(4.0)(108)} = 4.86 \text{ in.} < 7.5 \text{ in.}$$ [STD Art. 9.17.2]

Therefore, the depth of the compression block is less than flange thickness and the section must be considered as a rectangular section.

Design flexural strength of a rectangular section:

$$\phi M_n = \phi A^*_s f^*_{su}\, d\left(1 - 0.6\frac{\rho^* f^*_{su}}{f'_c}\right)$$ [STD Eq. 9-13]

where

ϕ = strength reduction factor = 1.0 [STD Art. 9.14]

M_n = nominal moment strength of a section

$$\phi M_n = 1.0(6.732)(265.0)(74.18)\left(1-0.6\frac{0.00084(265.0)}{4}\right)/12$$

= 10,660 ft-kips > M_u = 8,749 ft-kips O.K.

9.3.10 DUCTILITY LIMITS

9.3.10.1 Maximum Reinforcement

[STD Art. 9.18.1]

Pretensioned concrete members are designed so that the steel is yielding as ultimate capacity is approached.

Reinforcement index for rectangular section:

$$\rho^* \frac{f^*_{su}}{f'_c} < 0.36\beta_1 = 0.00084\left(\frac{265.0}{4}\right) = 0.0557 < 0.36(0.85) = 0.306 \text{ O.K.} \quad \text{[STD Eq. 9-20]}$$

9.3.10.2 Minimum Reinforcement

[STD Art. 9.18.2]

The total amount of pretensioned and non-pretensioned reinforcement should be adequate to develop an ultimate moment at the critical section at least 1.2 times the cracking moment, M^*_{cr}:

$$\phi\, M_n \geq 1.2\, M^*_{cr}$$

Compute cracking moment:

$$M^*_{cr} = (f_r + f_{pe})S_{bc} - M_{d/nc}\left(\frac{S_{bc}}{S_b} - 1\right) \quad \text{[STD Art. 9.18.2.1]}$$

where

f_r = modulus of rupture [STD Art. 9.15.2.3]

$$= 7.5\sqrt{f'_c} = 7.5\sqrt{6{,}500}\left(\frac{1}{1{,}000}\right) = 0.605 \text{ ksi}$$

f_{pe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads.

$$= \frac{P_{se}}{A} + \frac{P_{se}e_c}{S_b}$$

where

P_{se} = effective prestress force after losses = 1,023.9 kips

e_c = 30.78 in.

$$f_{pe} = \frac{1{,}023.9}{767} + \frac{1{,}023.9(30.78)}{14{,}915} = 1.335 + 2.113 = 3.448 \text{ ksi}$$

$M_{d/nc}$ = non-composite dead load moment at midspan due to self-weight of beam and weight of slab = 1,438.2 + 1,659.6 = 3,097.8 ft-kips

$$M^*_{cr} = (0.605 + 3.448)(20{,}090)\left(\frac{1}{12}\right) - 3{,}097.8\left(\frac{20{,}090}{14{,}915} - 1\right) = 5{,}711 \text{ ft-kips}$$

$1.2M^*_{cr}$ = 6,853 ft-kips < $\phi\, M_n$ = 10,660 ft-kips O.K.

Contrary to *LRFD Specifications,* the *Standard Specifications* indicate that this requirement must be satisfied only at the critical sections.

9.3.11 SHEAR DESIGN

[STD Art. 9.20]

Pretensioned members subject to shear should be designed so that:

$V_u \leq \phi\ (V_c + V_s)$ [STD Eq. 9-26]

where

V_u = the factored shear force at the section considered

V_c = the nominal shear strength provided by concrete

V_s = the nominal shear strength provided by web reinforcement

ϕ = strength reduction factor for shear = 0.90 [STD Art. 9.14]

The critical section in pretensioned concrete beams is located at a distance h/2 from the face of the support, according to the *Standard Specifications,* Article 9.20.1.4. In this example, the critical section for shear will be conservatively calculated from the centerline of support. The width of the bearing has not yet been determined, it will be conservatively assumed to be zero. Therefore, the detailed calculations are shown here for the section at (h_c /2= 80/2 = 40 in.) from the centerline of support. The following calculations demonstrate how to compute V_{ci} and V_{cw} at this location.

Compute V_{ci} :

$$V_{ci} = 0.6\sqrt{f'_c}\,b'd + V_d + \frac{V_i M_{cr}}{M_{max}}$$ [STD Eq. 9-27]

where

b' = width of web of a flanged member = 6.00 in.

f'_c = compressive strength of beam concrete at 28 days = 6,500 psi

V_d = total dead load at the section under consideration, (from **Table 9.3.4.1.2-1**)

= $V_g + V_S + V_b + V_{ws}$ = 45.3 + 52.2 + 5.7 + 11.3 = 114.5 kips

V_{LL+I} = unfactored shear force at section due to live load + impact = 63.6 kips

M_d = bending moment at section due to unfactored dead load

= 155.4 + 179.3 + 19.4 + 38.9 = 393.0 ft-kips

M_{LL+I} = live load bending moment plus impact = 211.5 ft-kips

V_u = factored shear force at the section

= $1.3(V_d + 1.67V_{LL+I})$ = 1.3[114.5 + 1.67 (63.6)] = 286.9 kips

M_u = factored bending moment at the section

= $1.3(M_d + 1.67M_{LL+I})$ = 1.3[393.0 + 1.67(211.5)] = 970.1 ft-kips

V_{mu} = factored shear force occurring simultaneously with M_u. Conservatively, use the maximum shear load occurring at this section.

= 1.3[114.5 + 1.67(63.6)] = 286.9 kips

M_{max} = maximum factored moment at section due to externally applied loads

= $M_u - M_d$ = 970.1 – 393.0 = 577.1 ft-kips

V_i = factored shear force at section due to externally applied loads occurring simultaneously with M_{max}

= $V_{mu} - V_d$ = 286.9 – 114.5 = 172.4 kips

f_{pe} = compressive stress in concrete due to effective pretension forces only (after allowance for all pretension losses), at extreme fiber of section where tensile stress is caused by externally applied loads.

The beam at this section is under positive flexure. Thus, f_{pe} should be evaluated at the bottom of the beam. Thus,

$$f_{pe} = \frac{P_{se}}{A} + \frac{P_{se}e}{S_b}$$

Compute eccentricity of the strands at $h_c/2$:

Center of gravity of 10 harped strands from the top of the beam:

$$6\text{ in.} + \frac{(72\text{ in.} - 12\text{ in.} - 6\text{ in.})}{48.5\text{ ft}}(3.33\text{ ft} + 0.5\text{ ft}) = 10.26\text{ in.}$$

Center of gravity of the 34 straight strands from the bottom of the beam = 4.00 in.

Center of gravity of all strands from the bottom of the beam at h/2:

$$\frac{10(72 - 10.26) + 34(4)}{44} = 17.12\text{ in.}$$

Therefore, the eccentricity of strand at h/2 = 36.60 − 17.12 = 19.48 in.

P_{se} = 1,023.9 kips

$$f_{pe} = \frac{1{,}023.9}{767} + \frac{1{,}023.9(19.48)}{14{,}915} = 1.335 + 1.337 = 2.672\text{ ksi}$$

f_d = stress due to unfactored dead load, at extreme fiber of section where tensile stress is caused by externally applied loads

$$= \left[\frac{(155.4 + 179.3)(12)}{14{,}915} + \frac{(19.4 + 38.9)(12)}{20{,}090}\right] = 0.304\text{ ksi}$$

M_{cr} = moment causing flexural cracking of section due to externally applied loads (after dead load)

$$= (6\sqrt{f'_c} + f_{pe} - f_d)S_{bc} \qquad \text{[STD Eq. 9-28]}$$

$$= \left(\frac{6\sqrt{6{,}500}}{1{,}000} + 2.673 - 0.304\right)\frac{20{,}090}{12} = 4{,}776\text{ ft-kips}$$

d = distance from extreme compressive fiber to centroid of pretensioned reinforcement. But d need not be taken less than 0.8 h_c = 64.00 in. [STD Art. 9.20.2.2]. The center of gravity of the pretensioned reinforcement is located at 17.12 in. from the bottom of the beam.

= 80 − 17.12 = 62.88 in. < 64.00 in.

Therefore, use d = 64.00 in.

$$V_{ci} = 0.6\sqrt{f'_c}\,b'd + V_d + \frac{V_i M_{cr}}{M_{max}} \qquad \text{[STD Eq. 9-27]}$$

$$= \frac{0.6\sqrt{6{,}500}\,(6.00)(64.00)}{1{,}000} + 114.5 + \frac{172.4(4{,}776)}{577.1} = 1{,}559.8\text{ kips}$$

This value should not be less than:

Minimum $V_{ci} = 1.7\sqrt{f'_c}\,b'd$ [STD Art. 9.20.2.2]

$$= \frac{1.7\sqrt{6{,}500}\,(6.00)(64.00)}{1{,}000} = 52.6 \text{ kips} < V_{ci} = 1{,}559.8 \text{ kips} \quad \text{O.K.}$$

Compute V_{cw}: [STD Art 9.20.2.3]

$$V_{cw} = \left(3.5\sqrt{f'_c} + 0.3\,f_{pc}\right)b'd + V_p \qquad \text{[STD Eq. 9-29]}$$

where

f_{pc} = compressive stress in concrete (after allowance for all pretension losses) at centroid of cross-section resisting externally applied loads. For a non composite section

$$f_{pc} = \frac{P_{se}}{A} - \frac{P_{se}e(y_{bc} - y_b)}{I} + \frac{(M_g + M_s)(y_{bc} - y_b)}{I}$$

$$= \frac{1{,}023.9}{767} - \frac{1{,}023.9(19.48)(54.77 - 36.60)}{545{,}894} + \frac{334.7(12)(54.77 - 36.60)}{545{,}894}$$

$$= 1.335 - 0.664 + 0.134 = 0.805 \text{ ksi}$$

V_p = vertical component of prestress force for harped strands

P_{se} = the effective prestress force for the harped strands

= 10(0.153)(152.1) = 232.7 kips

$V_p = P_{se}\sin\psi$ (see Section 9.3.7.5 for calculations of ψ)

= 232.7sin(5.30°) = 21.5 kips

$$V_{cw} = \left(\frac{3.5\sqrt{6{,}500}}{1{,}000} + 0.3(0.805)\right)(6.00)(64.00) + 21.5 = 222.6 \text{ kips}$$

The allowable nominal shear strength provided by concrete should be the lesser of V_{ci} (1,559.8 kips) and V_{cw} (222.6 kips). V_{cw} governs, so:

V_c = 222.6 kips

$V_u < \phi(V_c + V_s)$ (Eq. 9-26)

V_s = nominal shear strength provided by shear reinforcement

ϕ = strength reduction factor for shear = 0.90 [STD Art. 9.14]

$$\text{Required } V_s = \frac{V_u}{\phi} - V_c = \frac{286.9}{0.90} - 222.6 = 96.2 \text{ kips}$$

Calculate the maximum shear force that may be carried by reinforcement:

Maximum $V_s = 8\sqrt{f'_c}\,b'd$ [STD Art. 9.20.3.1]

$$= 8\sqrt{6{,}500}\,\frac{(6.00)(64.00)}{1{,}000} = 247.7 \text{ kips} > \text{required } V_s = 96.2 \text{ kips} \quad \text{O.K.}$$

Compute area of shear steel required [STD Art. 9.20.3.1]

$$V_s = \frac{A_v f_y d}{s} \quad \text{[STD Eq. 9-30]}$$

Solving for A_v:

$$A_v = \frac{V_s\, s}{f_y\, d}$$

where

A_v = area of web reinforcement, in.^{2}

s = longitudinal spacing of the web reinforcement, in.

Set s = 12 in. to have units of in.^{2}/ft for A_v.

$$A_v = \frac{(96.2)(12)}{(60)(64.00)} = 0.301 \text{ in.}^2\text{/ft}$$

Minimum shear reinforcement [STD Art. 9.20.3.3]

$A_{v\text{-min}}$ = minimum area of web reinforcement

$$A_{v\text{-min}} = \frac{50b's}{f_y} = \frac{50(6)(12)}{60{,}000} = 0.06 \text{ in.}^2\text{/ft} \quad \text{[STD Eq. 9-31]}$$

The required shear reinforcement is the maximum of A_v (0.301 in.^{2}/ft) and $A_{v\text{-min}}$ (0.06 in.^{2}/ft).

Maximum spacing of web reinforcement is 0.75 h_c or 24 in., unless V_s = 96.2 kips >

$$4\sqrt{f_c'}\,b'd = \frac{4\sqrt{6{,}500}\,(6.00)(64.00)}{1{,}000} = 123.8 \text{ kips} \quad \text{[STD Art. 9.20.3.2]}$$

Since V_s is less than the limit,

Maximum spacing = 0.75 h = 0.75 (72 + 7.5 + 0.5) = 60 in.

or = 24.00 in.

Therefore, maximum s = 24 in.

Use # 4, two-legged stirrups at 12 in. spacing (A_v = 0.40 in.^{2}/ft)

Note that the above calculations need to be repeated at regular intervals along the entire length of the beam to determine the area and spacing of shear reinforcement.

9.3.12 HORIZONTAL SHEAR DESIGN

[STD Art. 9.20.4]

The computation will be carried out for the section at a distance of h_c/2 from the centerline of support.

V_u = 286.9 kips

$$V_u \le \phi V_{nh} \quad \text{[STD Eq. 9-31a]}$$

where V_{nh} = nominal horizontal shear strength, kips

$$V_{nh} \ge \frac{V_u}{\phi} = \frac{286.9}{0.9} = 318.8 \text{ kips}$$

Case (a & b): Contact surface is roughened, or when minimum ties are used

[STD Art. 9.20.4.3]

Allowable shear force:

$V_{nh} = 80b_vd$

where

b_v = width of cross-section at the contact surface being investigated for horizontal shear = 42.00 in.

d = distance from extreme compressive fiber to centroid of the pretensioning force = 62.88 in.

Note: The full 'd' is used because the minimum value for d of 0.8 h_c does not apply to this calculation.

$$V_{nh} = \frac{80(42.00)(62.88)}{1{,}000} = 211.3 \text{ kips} < 318.8 \text{ kips} \quad \text{N.G.}$$

Case (c): Minimum ties provided, and contact surface roughened [STD Art. 9.20.4.3]

Allowable shear force:

$V_{nh} = 350b_vd$

$$= \frac{350(42.00)(62.88)}{1{,}000} = 924.3 \text{ kips} > 318.8 \text{ kips} \quad \text{O.K.}$$

Determine required stirrups for horizontal shear: [STD Art. 9.20.4.5]

$$\text{minimum } A_{vh} = 50\frac{b_v s}{f_y} = 50\frac{42(12)}{60{,}000} = 0.42 \text{ in.}^2/\text{ft}$$

The required minimum horizontal shear reinforcement, A_{vh} = 0.42 in.^{2}/ft is approximately equal to the vertical shear reinforcement provided, A_v = 0.40 in.^{2}/ft. O.K.

Maximum spacing = 4b = 4(6) = 24.00 in [STD Art. 9.20.4.5.a]

or = 24.00 in.

Therefore, maximum s = 24.00 in. > the provided s = 12 in.

9.3.13 PRETENSIONED ANCHORAGE ZONE

[STD Art. 9.22]

9.3.13.1 Minimum Vertical Reinforcement

In a pretensioned beam, vertical stirrups acting at a unit stress of 20,000 psi to resist at least 4% of the total pretensioning force, must be placed within the distance of d/4 of the beam end. [STD Art. 9.22.1]

Minimum stirrups at the each end of the beam:

P_s = prestress force before initial losses = 44(0.153)[(0.75)(270)] = 1,363.2 kips

4% P_s = 0.04(1,363.2) = 54.5 kips

$$\text{Required } A_v = \frac{54.5}{20} = 2.73 \text{ in.}^2$$

$$\frac{d}{4} = \frac{62.88}{4} = 15.75 \text{ in.}$$

Use 5 pairs of #5 @ 3 in. spacing at each end of the beam (A_v = 3.10 in.^{2})

9.3.13.2 Confinement Reinforcement

[STD Art. 9.22.2]

Provide nominal reinforcement to enclose the pretensioning steel for a distance from the end of the beam equal to the depth of the beam.

9.3.14 DEFLECTION AND CAMBER

9.3.14.1 Deflection Due to Pretensioning Force at Transfer

For harped strands:

$$\Delta_p = \frac{P_{si}}{E_{ci} I}\left(\frac{e_c L^2}{8} - \frac{e' a^2}{6}\right)$$

where

Δ_p = camber due prestress force at transfer, in.

P_{si} = total pretensioning force = 1,242.1 kips

I = moment of inertia of non-composite section = 545,894 in.^{4}

L = overall beam length = 121.0 ft

E_{ci} = modulus of elasticity of the beam concrete at release = 4,496 ksi

e_c = eccentricity of pretensioning force at the midspan = 30.78 in.

e' = difference between eccentricity of pretensioning steel at midspan and end
= $e_c - e_e$ = 30.78 − (36.60 − 18.09) = 12.27 in.

a = distance from the end of beam to harp point. = 48.50 ft

$$\Delta_p = \frac{1,242.1}{(4,496)(545,894)}\left(\frac{(30.78)[(121)(12)]^2}{8} - \frac{(12.27)[(48.5)(12)]^2}{6}\right) = 3.75 \text{ in.}\uparrow$$

9.3.14.2 Deflection Due to Beam Self-Weight

$$\Delta_{beam} = \frac{5 w_g L^4}{384\, E_{ci}\, I}$$

where w_g = beam weight = 0.799 kips/ft

Deflection due to beam self-weight at transfer (L = 121 ft):

$$\Delta_{beam} = \frac{5(0.799/12)[(121)(12)]^4}{(384)(4,496)(545,894)} = 1.57 \text{ in.}\downarrow$$

Deflection due to beam self-weight used to compute deflection at erection (L = 120 ft):

$$\Delta_{beam} = \frac{5(0.799/12)[(120)(12)]^4}{(384)(4,496)(545,894)} = 1.52 \text{ in.}\downarrow$$

9.3.14.3
Total Initial Deflection

Total deflection at transfer: 3.75 − 1.57 = 2.18 in.↑

Total deflection at erection, using PCI multipliers (see PCI Design Handbook)

$$1.80(3.75) - 1.85(1.520) = 3.94 \text{ in.}\uparrow$$

PCI multipliers used to calculate long-term deflection in building products have not proven to be accurate for bridge construction. Therefore, it is recommended that the designer follow the guidelines of the owner agency for whom the bridge is being designed. Frequently, no additional multipliers are used.

9.3.14.4
Deflection Due to Slab and Haunch Weights

$$\Delta_{slab} = \frac{5 w_s L^4}{384 E_c I}$$

where

w_s = slab weight + haunch weight = 0.922 kips/ft

E_c = modulus of elasticity of the beam concrete at service = 4,888 ksi

$$\Delta_{slab} = \frac{5(0.922/12)[(120)(12)]^4}{(384)(4,888)(545,894)} = 1.61 \text{ in.}\downarrow$$

9.3.14.5
Deflection Due to Barrier and Wearing Surface Weights

Since these loads are applied to the structure in its final location, L is the design span = 120 ft.

$$\Delta_{SDL} = \frac{5(w_b + w_{ws})L^4}{384 E_c I_c}$$

where

w_b = weight of barriers = 0.100 kips/ft

w_{ws} = weight of future wearing surface = 0.200 kips/ft

I_c = moment of inertia of composite section = 1,100,320 in.^{4}

$$\Delta_{SDL} = \frac{5\left(\frac{0.100 + 0.200}{12}\right)[(120)(12)]^4}{(384)(4,888)(1,100,320)} = 0.26 \text{ in.}\downarrow$$

9.3.14.6
Deflection Due to Live Load and Impact

Live load deflection limit (optional) = L/800

Δ_{max} = 120(12)/800 = 1.80 in.

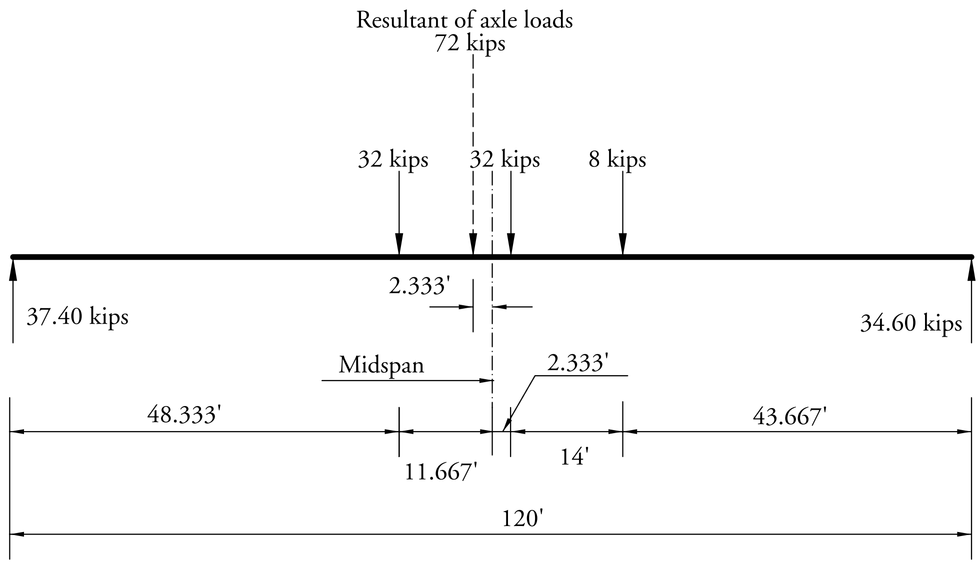
Some state DOTs consider that all beams act together in resisting deflection due to live load and impact. Using that assumption, the distribution factor for deflection is:

$$\frac{\text{number of lanes}}{\text{number of beams}} = \frac{4}{6} = 0.667 \text{ lanes / beam}$$

For all the design examples, live load deflection is conservatively computed based on the distribution factor used to calculate bending stresses.

To derive maximum moment and deflection at midspan due to the truck load, let the centerline of the beam coincide with the midpoint of the distance between the inner 32-kip axle and the resultant of the truck load (as shown in the **Figure 9.3.14.6-1**).

Figure 9.3.14.6-1
Design Truck Axle Load Positions on the Span for Maximum Moment



Beam analysis shows live load deflection at midspan is 0.80 in./lane.

$\Delta_{LL+I} = 0.80(1 + I)(DF_m)$

$\Delta_{LL+I} = 0.80(1.204)(0.818) = 0.79$ in. < 1.80 in.   O.K.